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THE OPERATION OF MAGNETIC AMPLIFIERS WITH VARIOUS TYPES OF LOADS

PART I - LOAD CURRENTS FOR GIVEN ANGLE OF FIRING

L. A. FINZI AND R. R. JACKSON

MAGNETIC AMPLIFIERS - TECHNICAL REPORT NO. 16

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Scope of the paper

The operation of magnetic amplifiers with sensitive core materials is characterized by time intervals in the cycle during which the voltage of the a.c. power source is balanced (or "absorbed") almost entirely by the rates of change of core fluxes, and by other intervals in which the core fluxes are nearly constant and the voltage of the power source substantially appears applied to the load.

Transitions from the first to the second mode are very abrupt. Thus load currents can be evaluated as if the voltage of the a.c. power source were periodically applied at some time or "angle of firing" by the firing of a thyratron or by the closing of a switch and were removed at some later time. A very simple solution is obtained if the output current upon firing is limited by nothing else but resistances. The analysis, however, becomes much more elaborate if this restriction is removed.

This paper examines instances of practical interest in which the load circuit includes resistances, inductances and also d.c. voltage sources, in conjunction with rectifying elements variously inserted. Load currents and gate winding currents are evaluated in terms of the angle of firing chosen as the independent variable, neglecting pre-firing currents in this approach. (A companion paper shall aim at the evaluation of the signal needed for wanted angles of firing in the various cases, with particular reference to some common types of high-gain amplifiers.)

PART I - LOAD CURRENTS FOR GIVEN ANGLE OF FIRING

1) Half-wave Amplifier Circuits

Figure 1-a represents the gate winding of a half-wave magnetic amplifier with a load circuit containing in series a resistance R_L, and inductance L_L and a d.c. voltage source E_L. (A circuit of this sort approximates e.g. the armature circuit of a separately excited d.c. motor at constant speed.) R_g is the resistance of the amplifier gate winding and gate rectifier and L_g is its saturated inductance. Thus the total resistance and inductance seen by the a.c. voltage source $v_g = V_{gm} \sin \omega t$ upon firing are $R = R_L + R_g$ and $L = L_L + L_g$. (If the rectifier has appreciable incremental resistance and threshold voltage in the forward direction, these effects can be accounted for by increasing R_g and modifying E_L.)

With $K = E_L / V_{gm}$ (where $-1 \leq K \leq 1$) the differential equation of the circuit in the saturated mode is simply

1-1) $$V_{gm} \left(\sin \omega t - K \right) = R i_L + L \frac{d i_L}{dt}$$

Thus for firing at $\omega t = \alpha$, under the assumption of negligible pre-firing magnetizing current

1-2) $$i_L = \frac{V_{gm}}{R} \left\{ \left[\cos\theta \sin(\omega t - \theta) - K \right] - \left[\cos\theta \sin(\alpha - \theta) - K \right] \epsilon^{-(\omega t - \alpha)\cot\theta} \right\}$$

where $$\tan\theta = \frac{\omega L}{R}$$

This expression is valid over the "saturation" interval initiated at α and terminated at the "extinction angle" β. At $\omega t = \beta$, i_L becomes zero. (The gate rectifier excludes the possibility of negative values for i_L.) Therefore the angle β is defined by the transcendental equation

1-3) $$0 = \left[\cos\theta \sin(\beta - \theta) - K \right] - \left[\cos\theta \sin(\alpha - \theta) - K \right] \epsilon^{-(\beta - \alpha)\cot\theta}$$

The cyclic average of output current is found as

1-4) $$I_{LS} = \frac{1}{2\pi} \int_{\alpha}^{\beta} i_L \, d\omega t = \frac{V_{gm}}{2\pi R} \left\{ \left(\cos\alpha - \cos\beta \right) - K(\beta - \alpha) \right\}$$

(where the additional subscript "s" is introduced to remind that contributions of pre-firing currents have been neglected).

The form of 1-4) appears well justified on the basis of "volt-time area" concepts. During the saturation interval the magnetic core does not absorb any voltage, and the voltage of the a.c. power supply is absorbed partly by the "battery" E_L and partly by the passive elements R and L. But the cyclic volt-radian area absorbed by L is zero (as $i_L = 0$ at $\omega t = \alpha$ and again at $\omega t = \beta$). Hence the volt-radian area $2\pi RI_{LS}$ absorbed by R results simply as the volt-radian area $V_{gm}(\cos\alpha - \cos\beta)$ of the power source minus the volt-radian area $E_L(\beta - \alpha)$ absorbed by the battery.

An explicit solution for β is not possible, although β can be evaluated from 1-3) e.g. by tedious cut-and-try methods. Even if a large number of systematic calculations are carried out the results, collected in graphical form, imply the presentation of families of families of curves, which make interpolations not too easy. (β depends on <u>three</u> arbitrary parameters, namely θ , K and α .)

Curves of this kind have been presented in the literature (References 1, 2 and 3) for certain ranges of the parameters. Nomographic procedures appear most helpful here, both in reducing the number of calculations and in presenting comprehensive results in a unified form which should allow for interpolations and should indicate clearly how changes of the various parameters affect the solution.

A nomogram of this kind (Ref. 4) -- derived in Appendix -- is presented in Figure 1-b. Given α on the proper θ -curve, the angle β is obtained on the same curve by interception with a straight line joining the given α point with the assigned K -point on the K -axis. Two examples are shown in dotted lines for $\theta = 60^0$ and $K = +0.3$ and $K = -0.3$; corresponding wave forms of load currents are seen in the oscillograms Figure 1-c and Figure 1-d. (The meaning of the x- and y-axes shown is discussed in the Appendix with additional comments. No use is made of the axes in the evaluation of β).

With the polarities and sign conventions chosen in Figure 1-a, K is positive when E_L opposes the flow of positive current. Situations may occur in which E_L has opposite polarity, i.e. aiding the flow of current. The solution given by the nomogram and by eq. 1-4) remains quite general, with the consideration that E_L and thus k are negative numbers in such cases. Instantaneous values of i_L are shown in Figure 1-d for situations of this kind.

With k sufficiently negative and L large enough it is possible that no extinction occurs. The amplifier core then remains saturated throughout the cycle, as no time is available for flux resetting. This possibility is recognized from the nomogram, as the straight line through k and α on the θ -curve will not yield any β intercept. In fact, the concept of firing loses meaning in such cases.

It may be noted that "pre-firing" core magnetizing currents have been neglected completely in the solution. This has been found expedient to simplify the integration of eq. 1-1). Corrections can be introducted evaluating the contribution I_{LP} given by these currents to the total load current $I_L = I_{LS} + I_{LP}$ whenever the

pre-firing currents are significant. E.g. the intervals in which the flux is swinging from or toward saturation are easily recognized in any type of operation, thus also the instantaneous magnetomotive force of the core is obtained from its dynamic loop. The contributions given by the gate winding current to this total magnetomotive force depend on the type of circuitry used for resetting (i.e. for controlling α) and are readily evaluated in any specific situation. In view of the large variety of means by which the firing of half-wave amplifiers is controlled in modern practice it appears unwise to enter into details.

2) Half-wave Amplifiers with Load Discharging Paths

In Figure 2-a the load is shunted by a "discharging" or "freewheeling" rectifier. Since $i_L = i_g + i_d$ and i_d cannot be negative, it follows that the load current i_L cannot be less than the winding current i_g, though at some times it may well become larger. Depending on the parameters of the circuit various possibilities arise in the operation.

A) The flow of current through the load is discontinuous (Figure 2-b)

that is, i_L decays to zero at some time between successive firings. Since at $\omega t = \alpha$ the load current is zero, eq. 1-2) describes the initial transient upon firing. The discharging rectifier does not conduct yet and the load current equals the winding current. This is a first "forcing" part of the transient. (A prime superscript will be used for currents during this interval.)

However, at some angle γ the load voltage $v_L = R_L i_L' + L_L \frac{di_L'}{dt} + E_L$ becomes zero trying to reverse; the discharging rectifier becomes conducting, short-circuiting the load and a new "relaxation" interval begins. The angle γ is obtained by solving the transcendental equation

2-1) $$V_{gm} \sin \gamma = \left(R_g i_L' + L_g \frac{di_L'}{dt} \right)_{t = \frac{\gamma}{\omega}}$$

(In most practical cases $\gamma \cong \pi$, that is, relaxation begins when the a.c. supply voltage is about zero.)

In the relaxation interval (for which double prime superscripts will be used) the load current i_L'' is defined by the differential equation

2-2) $$0 = R_L i_L'' + L_L \frac{di_L''}{d(t-\gamma/\omega)} + E_L$$ with the condition $(i_L'')_{\omega t=\gamma} = (i_L')_{\omega t=\gamma}$ while

2-3) $$V_{gm} \sin \omega t = R_g i_g'' + L_g \frac{di_g''}{d(t-\gamma/\omega)}$$

describes the current flowing in the circuit of source and gate winding, by-passing the load through the now available discharging rectifier path. This current drops to zero rather rapidly and then the gate rectifier stops its further flow.

Integration of eq. 2-2) yields

2-4) $$i_L'' = \left(i_{L\gamma}' + \frac{E_L}{R_L}\right)\varepsilon^{-\frac{R_L}{\omega L_L}(\omega t-\gamma)} - \frac{E_L}{R_L}$$

At some angle $\omega t = \delta$, i_L'' becomes zero, and relaxation terminates.

The approximation $\gamma \cong \pi$ allows ready calculation of $i_{L\gamma}'$ and δ . Namely

2-5) $$i_{L\gamma}' \cong i_{L\pi}' = \frac{V_{gm}}{R}\left\{\cos\theta\sin\theta - \cos\theta\sin(\alpha-\theta)\varepsilon^{-(\pi-\alpha)\cot\theta} - K\left(1-\varepsilon^{-(\pi-\alpha)\cot\theta}\right)\right\}$$

$$= \frac{V_{gm}}{R}\left\{a - Kb\right\}$$

(where a and b are both functions of θ and α , plotted for the sake of convenience in the family of curves of Figure 3), and

2-6) $$\delta \cong \frac{\omega L_L}{R_L}\ln\left(\frac{R_L i_{L\pi}'}{E_L}+1\right) + \pi$$

The cyclic average current is obtained as the sum

2-7) $I_{LS} = I_{LS}' + I_{LS}''$ where

2-8) $$I_{LS}' = \frac{1}{2\pi}\int_\alpha^\pi i_L'\,d\omega t = \left\{V_{gm}(\cos\alpha - \cos\pi) - E_L(\pi-\alpha) - Li_{L\pi}'\right\}/2\pi R$$

2-9) $$I_{LS}'' = \frac{1}{2\pi}\int_\pi^\delta i_L''\,d\omega t = \left\{L_L i_{L\pi}' - E_L(\delta-\pi)\right\}/2\pi R$$

These expressions indicate that the volt-radian area of the a.c. source, not absorbed by the core during the forcing interval from α to π, equals the volt-radian area $2\pi R I_{LS}'$ plus the volt-radian area $E_L(\pi-\alpha)$ absorbed by the "battery", plus volt-radian area $L i_{L\pi}'$ absorbed by the circuit inductances.

In the second part of the transient the volt-radian area $L_L i'_{L\pi}$ of the load inductance is converted into the volt-radian area $2\pi R_L I''_{Ls}$ plus the volt-radian area $E(\delta - \pi)$ further absorbed by the battery over the interval π to δ.

If the resistance and saturated inductance of the gate winding are neglected 2-7) simplifies into

2-10) $$I_{Ls} = \frac{V_{gm}}{2\pi R}\left\{(1+\cos\alpha) - K(\delta-\alpha)\right\}$$

It may be noted that for certain angles of firing, relaxation may not take place at all during the cycle if E_L is positive and sufficiently large. The occurrence of this sub-case may be recognized from the nomogram of Figure 1-b whenever this yields $\beta < \pi$. Then eq. 1-2) and eq. 1-4) apply throughout.

B) <u>The flow of current through the load is continuous</u> (Figure 2-c)

In many instances (e.g. zero or negative values of E_L or large inductances or early firing in the cycle) eq. 2-6) of the preceding analysis yields $\delta > \alpha + 2\pi$, that is, a new firing would seem to occur before the load current of the preceding relaxation interval has died out, in contradiction with the initial condition of zero load current used in the integration of eq. 1-1). In this case the differential eq. 1-1) describes still further the forcing part of the transient but the solution is modified by the existence of an initial current $i'_{L\alpha} > 0$.

In order to obtain workable results it appears expedient to neglect the resistance R_g and the saturated inductance L_g of the amplifier winding. As a first consequence of this approximation i_g reaches immediately the value $i'_{L\alpha}$ upon core saturation. Thus eq. 1-2) modifies into

2-11) $$i'_L = \frac{V_{gm}}{R}\left\{[\cos\theta\sin(\omega t-\theta) - K] - [\cos\theta\sin(\alpha-\theta) - K]\,\varepsilon^{-(\omega t-\alpha)\cot\theta}\right\} + i'_{L\alpha}\,\varepsilon^{-(\omega t-\alpha)\cot\theta}$$

Eq. 2-11) is valid from α until π. At $\omega t = \pi$ the discharging rectifier becomes conducting and eq. 2-4) (with $\gamma = \pi$) describes the relaxation interval, which lasts until $\omega t = \alpha + 2\pi$.

The steady-state condition $i''_{L(\alpha+2\pi)} = i'_{L(\alpha)}$ offers the key for the complete evaluation of instantaneous load currents in terms of load parameters and angle of firing. In fact, having neglected R_g and L_g,

2-12) $$i''_L = \left(i'_{L\alpha} + \frac{E_L}{R_L}\right)\varepsilon^{-[\omega t-(\alpha+2\pi)]\cot\theta} - \frac{E_L}{R_L}$$ where

2-13) $$i'_{L\alpha} = \frac{V_{gm}}{R}\left\{\frac{\cos\theta\sin\theta\,\varepsilon^{-(\alpha+\pi)\cot\theta} - \cos\theta\sin(\alpha-\theta)\,\varepsilon^{-2\pi\cot\theta}}{1-\varepsilon^{-2\pi\cot\theta}} - K\right\}$$

Eqs. 2-11) and 2-12) yield the instantaneous current in forcing and relaxation intervals in terms of $i'_{L\alpha}$, given by 2-13). The cyclic average of load current is

$$I_{LS} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i'_L \, d\omega t + \frac{1}{2\pi}\int_{\pi}^{\alpha+2\pi} i'_L \, d\omega t = I'_L + I''_L = \frac{V_{gm}}{2\pi R}\left\{(1+\cos\alpha) - 2\pi K\right\} \tag{2-14}$$

This expression is obviously obtainable simply from volt-time area considerations. Eq. 2-14) together with the recognition that the core is saturated only from α to π , supplies sufficient information for relating cyclic average output to control in most common half-wave amplifier circuits. On the other hand, eqs. 2-11) to 2-13) are needed if the wave form of load current is of interest.

3) Two-core Amplifiers with a.c. Output

The analysis of Section 1 is extended readily to resistive-inductive loads supplied directly by two-core amplifiers with a.c. output (e.g. doubler or external feedback amplifiers). No need is felt here to neglect amplifier resistances and saturated inductances, that is, R and L result again from the combination of the load components R_L and L_L in series with the properly evaluated resistance and saturated inductance R_a and L_a of whichever amplifier circuit delivers the output current. For the sake of generality, a d.c. voltage source E_L can be included in the load, if this source is contained within a rectifier bridge of its own to constantly oppose the flow of load current in alternating half-cycles, as shown in Figure 4-a.

The instantaneous output current $i_G = i_L$ is expressed by eq. 1-2). The angle of extinction β is found from the nomogram of Figure 1-b (with $K \geq 0$ for the type of load shown in Figure 4-a). Thus the wave form of load current (Figure 4-b) is described and its half-cyclic average (as well as its rms value and also the power output) can be evaluated. In fact, the half-cyclic average is simply

$$I_{LS} = \frac{1}{\pi}\int_{\alpha}^{\beta} i_L \, d\omega t = \frac{V_{gm}}{\pi R}\left\{(\cos\alpha - \cos\beta) - K(\beta-\alpha)\right\} \tag{3-1}$$

Since firing occurs at α and then again at $\alpha+\pi$ it is evident that α cannot be advanced in the half-cycle below a value α^* for which $\beta = \alpha^* + \pi$. In this limiting condition (which can be easily determined for given K and θ on the nomogram by pivoting a straight edge about K) the amplifier does not absorb the a.c. supply voltage at anytime. The half-cyclic load current is maximum and is

$$I_{L\,max} = \frac{V_{gm}}{\pi R}\left\{2\cos\alpha^* - K\pi\right\} \tag{3-2}$$

If $K = 0$, eq. 1-3) yields $\alpha^* = \theta$; and $I_{L\,max} = \frac{2}{\pi}\frac{V_{gm}}{\sqrt{R^2+\omega^2L^2}}$

On the other hand, the analysis of Section 1 can be extended to some two-core situations in which it may be said that a negative k appears in the output circuit. This occurs in simple saturable reactors (e.g. with series connected gate windings, as obtained from Figure 8-a under elimination of the feedback windings) as the control voltage E_c as well as the total control circuit resistance R_c and inductance L_c appear reflected into the gate circuit by the "current transformer" action of the core which is unsaturated. With the initial sign conventions, k here is negative as E_c is reflected with polarity aiding the flow of current in either saturation interval in the cycle.

Namely β is found from the nomogram of Figure 1-b with $k = -\frac{|E_c|}{V_{gm}}\frac{N_g}{N_c}$.
R and L are the resistances and inductances of the output circuit modified by addition of the reflected resistances and inductances of the control circuit. (More precisely, R_a is then the resistance of both gate windings plus $R_c\ N_g^2/N_c^2$; L_a is four times the saturated inductance of one gate winding plus conceivable additional inductances of the control circuit multiplied times N_g^2/N_c^2.) Thus the wave forms of currents, as obtained by others (Reference 4), are defined completely (Figure 4-c) and the influence of the various parameters of gate and control is evidenced. In particular, eqs. 3-1) and 3-2) still apply.

(Moderate extensions of this treatment can be made if feedback turns are added to the saturable reactor, as shown in Figure 8-a, under consideration that the transformer ratio applied in the reflection of control circuit quantities becomes $(N_g - N_f)/N_c$. Phenomena of commutation within the feedback loop limit this extension to cases of comparatively low control circuit impedances.)

In Figures 4-b and 4-c the load current reaches its peak value at ξ. Knowledge of this angle is of interest in some situations; as shall be shown in the companion paper, Part II, this is the case, for instance, in the evaluation of control current requirements for certain types of two-core amplifiers. Graphical means for a quick determination of ξ are given in the Appendix.

4) Two-core Amplifiers with a.c. Output and Externally Rectified Load

Unidirectional load currents are obtained from magnetic amplifiers with a.c. output by enclosing the whole load (resistive-inductive, with d.c. voltage sources) within a rectifier bridge, (Figure 5-a). In this case the instantaneous load current cannot be less than the absolute value of the amplifier output current, though at times it may well become larger, as discharging paths are offered by the four arms of the bridge in simultaneous conduction. The analyses of Section 2 are modified here under the recognition that firing occurs at α and then again at $\alpha + \pi$.

The analysis of case A) of discontinuous flow applies as long as eq. 2-6) yields load current extinction at $\delta \leq \alpha + \pi$. The instantaneous currents i'_L and i''_L during forcing and relaxation are still expressed by eqs. 1-2) and 2-4) and shown in Figure 5-b. Also, the current $i'_{L\pi}$ at the instant of transition from forcing to relaxation is given by eq. 2-5).

But as two firings take place now in one cycle of the power supply, the average load I_L is evaluated now as $I_{LS} = I'_L + I''_L$ where I'_L and I''_L are twice the values given by eqs. 2-8) and 2-9). If resistances R_a and inductances L_a of gate windings (accounting also for reflected quantities from other coupled amplifier circuits) are negligible,

4-1) $$I_{LS} = \frac{V_{gm}}{\pi R}\left\{(1+\cos\alpha) - k(\delta-\alpha)\right\}$$

On the other hand, if eq. 2-6) yields $\delta > \alpha + \pi$, the analysis of case B) of continuous flow applies, with the expedient approximation $R_a = L_a = 0$. The matching condition of equal load currents at two successive firings is stated here as $i''_{L(\alpha+\pi)} = i'_{L\alpha}$.

Thus the instantaneous load currents during forcing and relaxation (as shown in Figure 5-c are

2-11) $$i'_L = \frac{V_{gm}}{R}\left\{\left[\cos\theta\sin(\omega t-\theta)-k\right]-\left[\cos\theta\sin(\alpha-\theta)-k\right]\varepsilon^{-(\omega t-\alpha)\cot\theta}\right\} + i'_{L\alpha}\varepsilon^{-(\omega t-\alpha)\cot\theta}$$

and 4-2) $$i''_L = \left(i'_{L\alpha} + \frac{E_L}{R_L}\right)\varepsilon^{-[\omega t-(\alpha+\pi)]\cot\theta} - \frac{E_L}{R_L}$$ where

4-3) $$i'_{L\alpha} = \frac{V_{gm}}{R}\left\{\frac{\cos\theta\sin\theta\,\varepsilon^{-\alpha\cot\theta} - \cos\theta\sin(\alpha-\theta)\,\varepsilon^{-\pi\cot\theta}}{1-\varepsilon^{-\pi\cot\theta}} - k\right\} = \frac{V_{gm}}{R}(C-k)$$

The contributions given by the forcing and relaxation intervals to the average load current I_L are

4-4) $$I'_{LS} = \frac{1}{\pi}\int_{\alpha}^{\pi} i'_L\,d\omega t = \frac{V_{gm}(1+\cos\alpha) - L\left(i'_{L\alpha}+\frac{E_L}{R_L}\right)\left(\varepsilon^{\alpha\cot\theta}-1\right) - E_L(\pi-\alpha)}{\pi R_L}$$

4-5) $$I''_{LS} = \frac{1}{\pi}\int_{\pi}^{\alpha+\pi} i''_L\,d\omega t = \frac{L\left(i'_{L\alpha}+\frac{E_L}{R_L}\right)\left(\varepsilon^{\alpha\cot\theta}-1\right) - E_L\alpha}{\pi R_L}$$ and thus

4-6) $$I_{LS} = I'_{LS} + I''_{LS} = \frac{V_{gm}}{\pi R_L}\left[(1+\cos\alpha) - k\pi\right]$$ (where k is a negative number if E_L aids the flow of load current).

Eq. 4-6) is very simple and is obtainable directly from volt-time area considerations. However, in many two-core amplifiers, the evaluation of the signal current needed for a wanted I_L depends on the knowledge of gate winding currents during saturation intervals. These saturation intervals coincide with the forcing intervals, during which $i'_L = i_G$. This implies that there is need here for calculations of $i'_{L\alpha}$, as given by eq. 4-3) A family of curves expressing C in terms of θ and α is presented in Figure 6 for the sake of convenience.

In fact, for the evaluation of signal currents it may be of interest in some cases to know the value $i'_{L\pi}$ of load current at the transition from forcing to relaxation. In this situation of continuous flow for the two-core amplifier, it is found

$$4\text{-}7) \qquad i'_{L\pi} = \frac{V_{gm}}{R}\left\{ \frac{\cos\theta\sin\theta - \cos\theta\sin(\alpha-\theta)\,\varepsilon^{-(\pi-\alpha)\cot\theta}}{1-\varepsilon^{-\pi\cot\theta}} - K \right\}$$

$$= \frac{V_{gm}}{R}\,(d-K)$$

A family of curves expressing d in terms of θ and α is presented in Figure 7.

5) Two-core Amplifiers with d.c. Output and Load Discharging Provisions (Fig.8-a,c,d)

Situations of this sort occur if the load is supplied by the output of a "center tap" amplifier, or also if it is inserted in series with the N_f coils in rectified feedback circuit of "external feedback" amplifiers. In either case, for proper operation, a discharging rectifier is connected directly across the load to offer a relaxation path, while unidirectional current is supplied from the power source to the load during forcing only. All analyses and results of the preceding Section 4 apply without modifications. The same analysis applies also if the load is in the output circuit of a "bridge-type" amplifier where a relaxation path short-circuiting the load is offered by the two lower rectifiers in series (Figure 8-d).

6) Two-core Amplifiers with d.c. Output without Load Discharging Provisions

If the load discharging rectifier is removed in the center-tap amplifier of Figure 8-c, the load current finds relaxation paths through both gate coils N_g. As relaxation initiates, upon reversal of the a.c. supply voltage, this voltage is balanced by the rate of change of both core fluxes, while comparatively large currents are induced in the control circuit by transformer action to cancel the magnetomotive forces of the relaxation currents.

A similar situation occurs in the external feedback of Figure 8-a if the load is inserted in the feedback loop without providing a load discharging rectifier directly across the load. The relaxation current finds then a path within the loop formed by the N_f coils on both cores through the four rectifiers of the bridge in simultaneous conduction, and comparatively large currents again are induced in the control circuit.

The implications of these undesirable types of relaxation in terms of amplifier control are examined in Section 8 of the companion paper, Part II. But the considerations given in Section 4 of this paper still apply to the determination of the load current for any assumed angle of firing α, at least as long as the resistances of these relaxation paths and reflected circuits do not affect remarkably the relaxation decay.

7) Conclusions

An analysis of load circuit behavior upon firing is presented for certain types of loads which are of most actual interest in magnetic amplifier applications. This is substantially a treatment of certain linear networks which contain rectifying elements and voltage sources and are submitted to recurrent switching transients. Nomograms and charts are given which eliminate the need for numerical cut-and-try solutions of transcendental equations appearing in problems of this kind.

For non-rectified loads a quick determination of the angle of extinction, β, is made possible in terms of the constant of the circuit and of the angle of firing, α, taken as the independent variable. Knowledge of the angle of extinction is indispensable for the evaluation of the output current and for the determination of the saturation interval. (This, in turn, is essential for the further determination of the corresponding amplifier signal requirements.)

For rectified loads means are given to evaluate instantaneous or average load currents separately during forcing and relaxation intervals. (Again this is essential also for the further determination of the corresponding signal.)

The further question of the evaluation of the signal required for any wanted angle of firing is treated in the companion paper, Part II, for the same types of loads, with particular reference to certain kinds of two-core amplifiers with sensitive core materials.

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APPENDIX

A) Derivation of the nomogram of Figure 1-b.

Eq. 1-3) is rewritten:

A-1) $$\left\{\cos\theta \sin(\alpha-\theta)-K\right\}\varepsilon^{\alpha\cot\theta} = \left\{\cos\theta \sin(\beta-\theta)-K\right\}\varepsilon^{\beta\cot\theta},$$

where $0\leq\theta\leq\pi/2,\quad 0\leq\alpha\leq\pi,\quad -1\leq K\leq 1,\quad \alpha<\beta$

Or,

A-2) $$F(\alpha,\theta,K) = F(\beta,\theta,K) = Z$$ and

A-3) $$F(\alpha,\theta,K)-Z=0,$$ A-4) $$F(\beta,\theta,K)-Z=0$$

Eq. A-3) can be rewritten as the form

$$\begin{vmatrix} 1 & 0 & Z \\ 0 & 1 & K \\ 1 & \varepsilon^{\alpha\cot\theta} & \varepsilon^{\alpha\cot\theta}\cos\theta\sin(\alpha-\theta) \end{vmatrix} = 0,$$ or, after manipulation:

$$\text{A-5)} \quad \begin{vmatrix} 0 & Z & 1 \\ 1 & k & 1 \\ \dfrac{\varepsilon^{\alpha \cot\theta}}{1+\varepsilon^{\alpha\cot\theta}} & \dfrac{\varepsilon^{\alpha\cot\theta}\cos\theta\sin(\alpha-\theta)}{1+\varepsilon^{\alpha\cot\theta}} & 1 \end{vmatrix} = 0 .$$

In a system of orthogonal x, y coordinates, a family of curves for constant θ 's is plotted with

$$x = \frac{\varepsilon^{\alpha\cot\theta}}{1+\varepsilon^{\alpha\cot\theta}} \qquad \text{and} \qquad y = \frac{\varepsilon^{\alpha\cot\theta}(\cos\theta\sin(\alpha-\theta))}{1+\varepsilon^{\alpha\cot\theta}} .$$

One such curve is shown in Figure 9. Moreover, a line $x = 1$, $y = k$ and a line $x = 0$, $y = Z$ are plotted. Thus a straight line through the point α on the θ -curve and through k on the k-axis locates on the Z-axis the value of Z satisfying eq. A-3). In view of the identity of eqs. A-3) and A-4) the same straight line intersects the θ -curve at point β which satisfies eq. A-4) for the same θ, k and Z. That is, given α, θ, and k, the corresponding angle of extinction β, defined by eq. A-1), is found by alignment, drawing a straight line through the points (α, θ) on the proper θ -curve, and the point k on the k-axis. β is determined by the intersection of this line with the same θ -curve, so that the Z-axis is not needed for this determination. (Thus no Z-axis is shown in Figure 1-b, for the purpose of better utilization of the over-all dimensions available for this drawing.)

It may be noted that no β intersection may be found for certain sets of α, θ and negative k values. This is the case when no extinction occurs in the cycle; that is, the reactor is saturated throughout.

On the other hand, more than one β intersection may be given on the θ -curve by the straight line; the proper solution is given then by the lowest of the β values located.

The determination of β on the nomogram presented is awkward for $\theta < 30^\circ$. It would be possible to expand somewhat this region of the nomogram by projective geometry transformations, but this would result in lack of clarity in other more important regions. In fact, as θ approaches zero, the current i_L assumes very the form

$$i_L = \frac{V g_m}{R}\left(\cos\theta\sin(\omega t-\theta) - k\right), \qquad \text{and}$$

$$\beta \simeq \sin^{-1}\left(\frac{k}{\cos\theta}\right) + \theta$$

B) Evaluation of the angle ξ of peak output current.

As mentioned in the text, practical need is felt in some instances for a determination of the angle ξ at which the amplifier output current reaches its peak. This determination is accomplished by means given below:

Case I) Two-core amplifiers with a.c. output or with rectified output in discontinuous flow. (Sections 3, 4, 5 and 6).

In either situation the instantaneous load current initiating at rises toward its crest as per eq. 1-2). By maximizing i_L in this equation, the following relation results:

$$\text{B-1)} \qquad \sin\theta\cos(\xi-\theta)\,\varepsilon^{\xi\cot\theta} = -\left\{\cos\theta\sin(\alpha-\theta)-k\right\}\varepsilon^{\alpha\cot\theta}$$

To solve this transcendental equation for ξ let's define

$$P = \sin\theta\cos(\xi-\theta)\,\varepsilon^{\xi\cot\theta}$$

A family of curves expressing ξ for various values of P and θ is presented in Figure 10. On the other hand, from eq. A-1) it is seen that P coincides in this case with the negative of the quantity Z already defined in eq. A-2). Now Z is obtained graphically for given values of α, θ and k, as shown in Figure 9. Thus use of the nomogram of Figure 1-b, extended in size to include the Z-axis in conjunction with the curves of Figure 10, enables determination of ξ for any set of α, θ and k.

Case II) Two-core amplifiers with rectified output in continuous flow. (Sections 4, 5 and 6).

Upon firing the instantaneous load current rises from its initial value $i'_{L\alpha}$ towards its crest, as per eq. 2-11), where $i'_{L\alpha}$ is given by eq. 4-5). The current i'_L reaches its maximum at the angle ξ defined by

$$\text{B-2)} \qquad \sin\theta\cos(\xi-\theta)\,\varepsilon^{\xi\cot\theta} = \frac{\cos\theta\sin\theta - \cos\theta\sin(\alpha-\theta)\,\varepsilon^{\alpha\cot\theta}}{1-\varepsilon^{-\pi\cot\theta}}$$

The quantity $P = \sin\theta\cos(\xi-\theta)\varepsilon^{\xi\cot\theta}$ in this case cannot be obtained simply from the nomogram. Thus curves are presented in Figure 11 expressing the right hand side of eq. B-2) in terms of α and θ. For any set of α and θ, P is evaluated from these curves. Use of this value of P in conjunction with the curves of Figure 10 yields ξ.

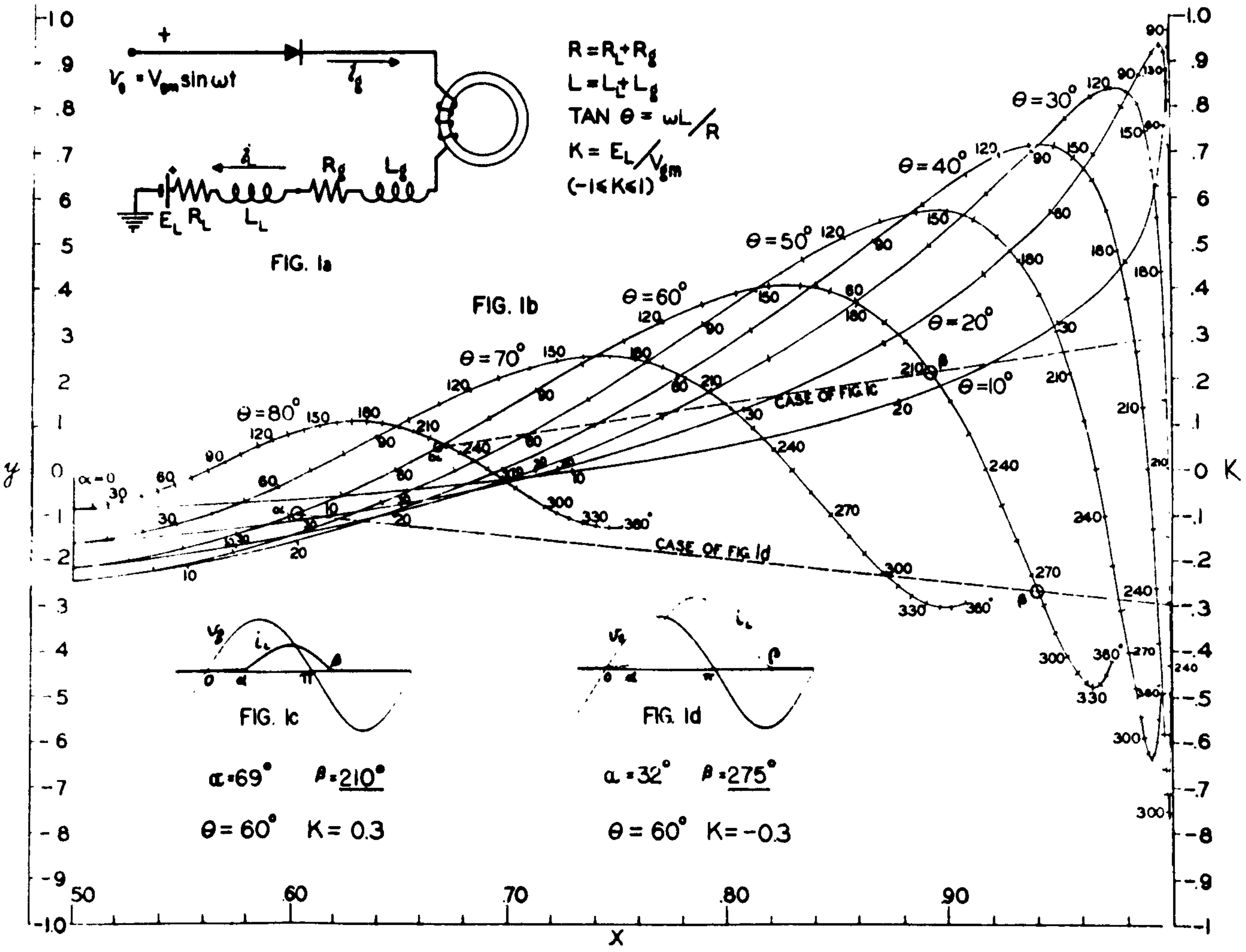

Fig. 1 Half-wave amplifier circuit - Determination of the angle of extinction.

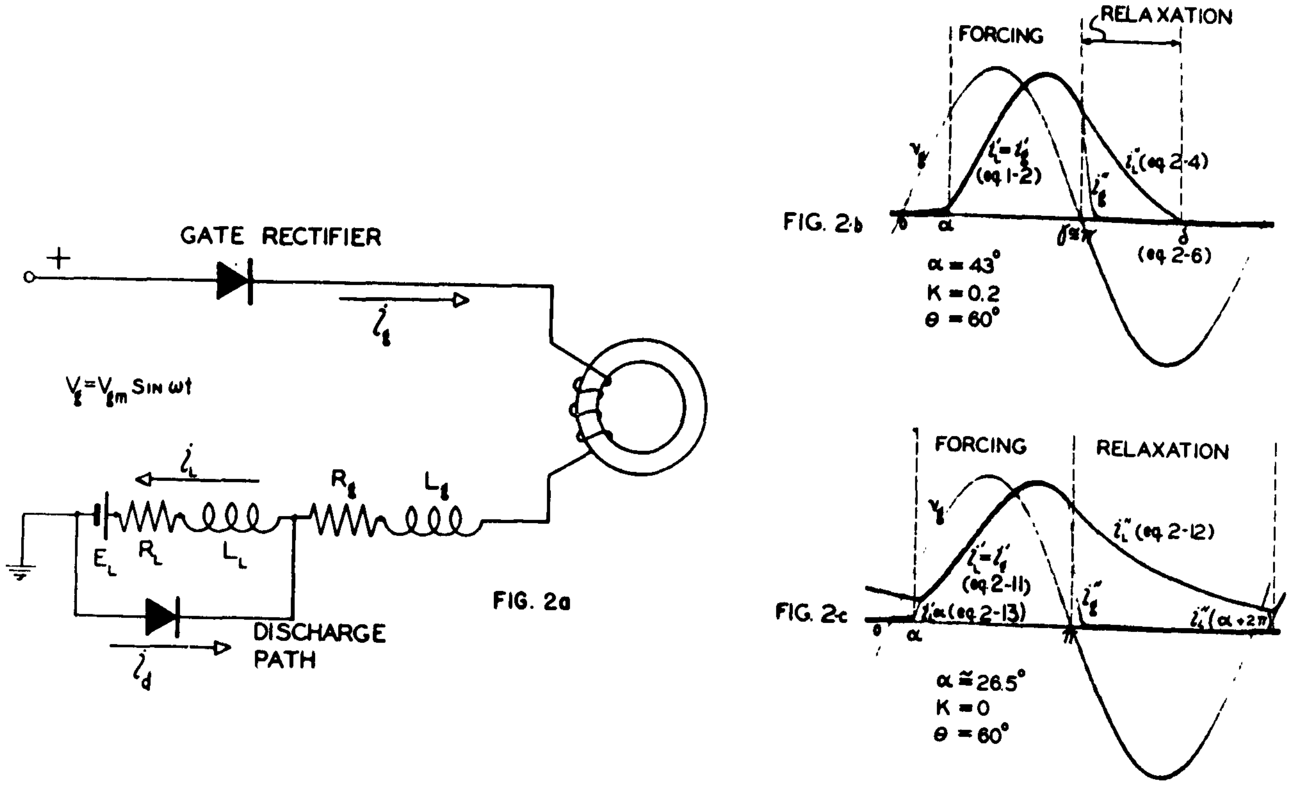

Fig. 2 (a) Half-wave amplifier with load discharging path
(b) Load current, discontinuous flow
(c) Load current, continuous flow

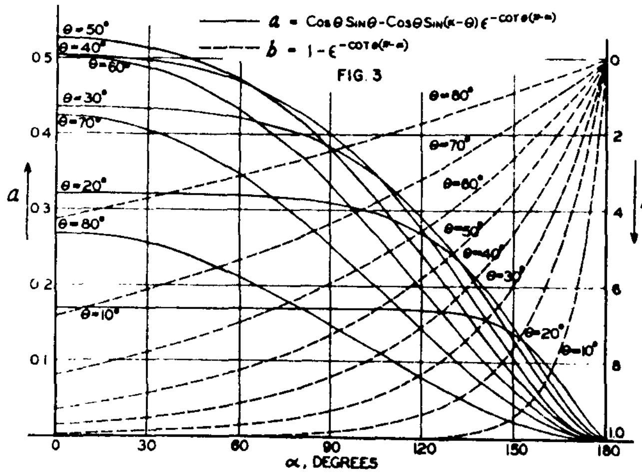

Fig. 3 The quantities a and b of equation (2-5)

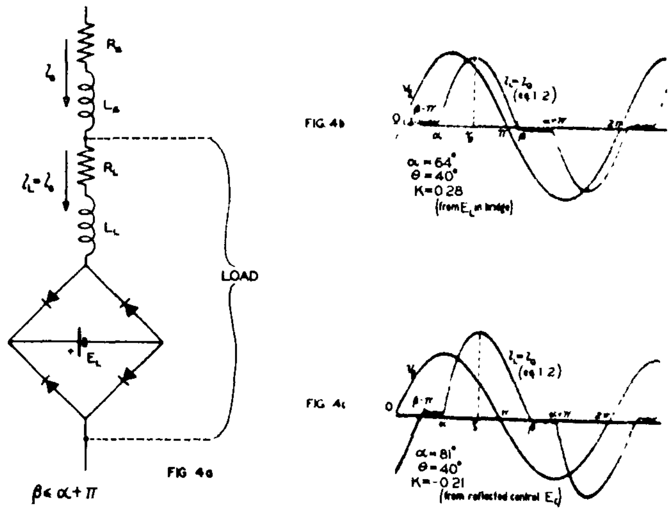

Fig. 4 Two-core amplifiers with a.c. output; non-rectified load inductance.

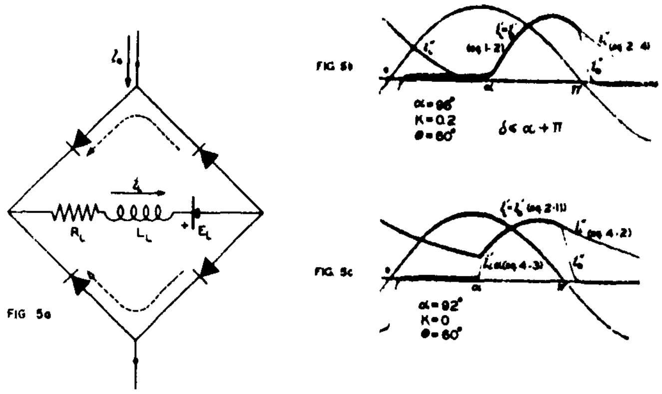

Fig. 5 Two-core amplifiers - Rectified load

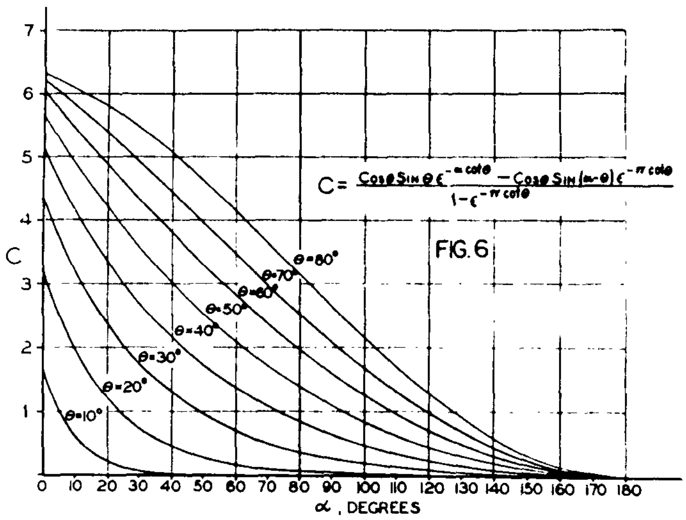

Fig. 6 The quantity c of eq. (4-3)

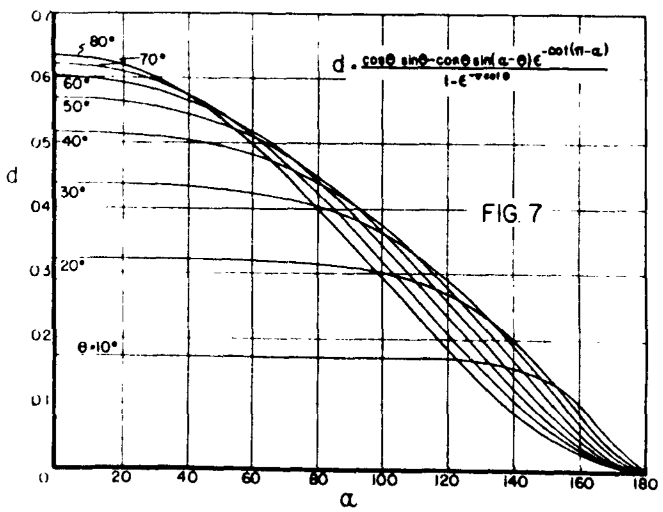

Fig. 7 The quantity d of eq. (4-7)

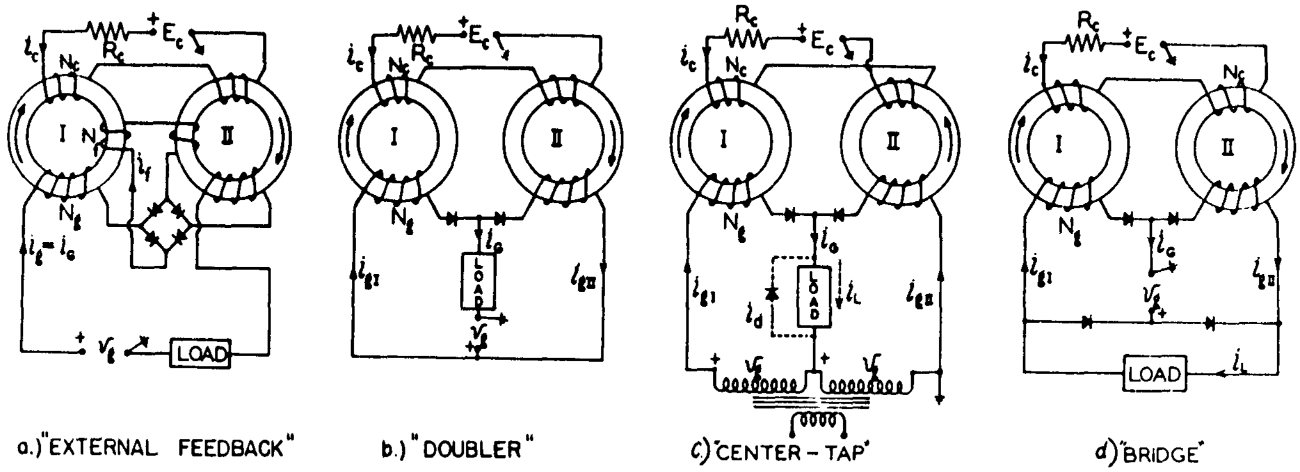

FIG. 8

Fig. 8 Basic two-core amplifier circuits

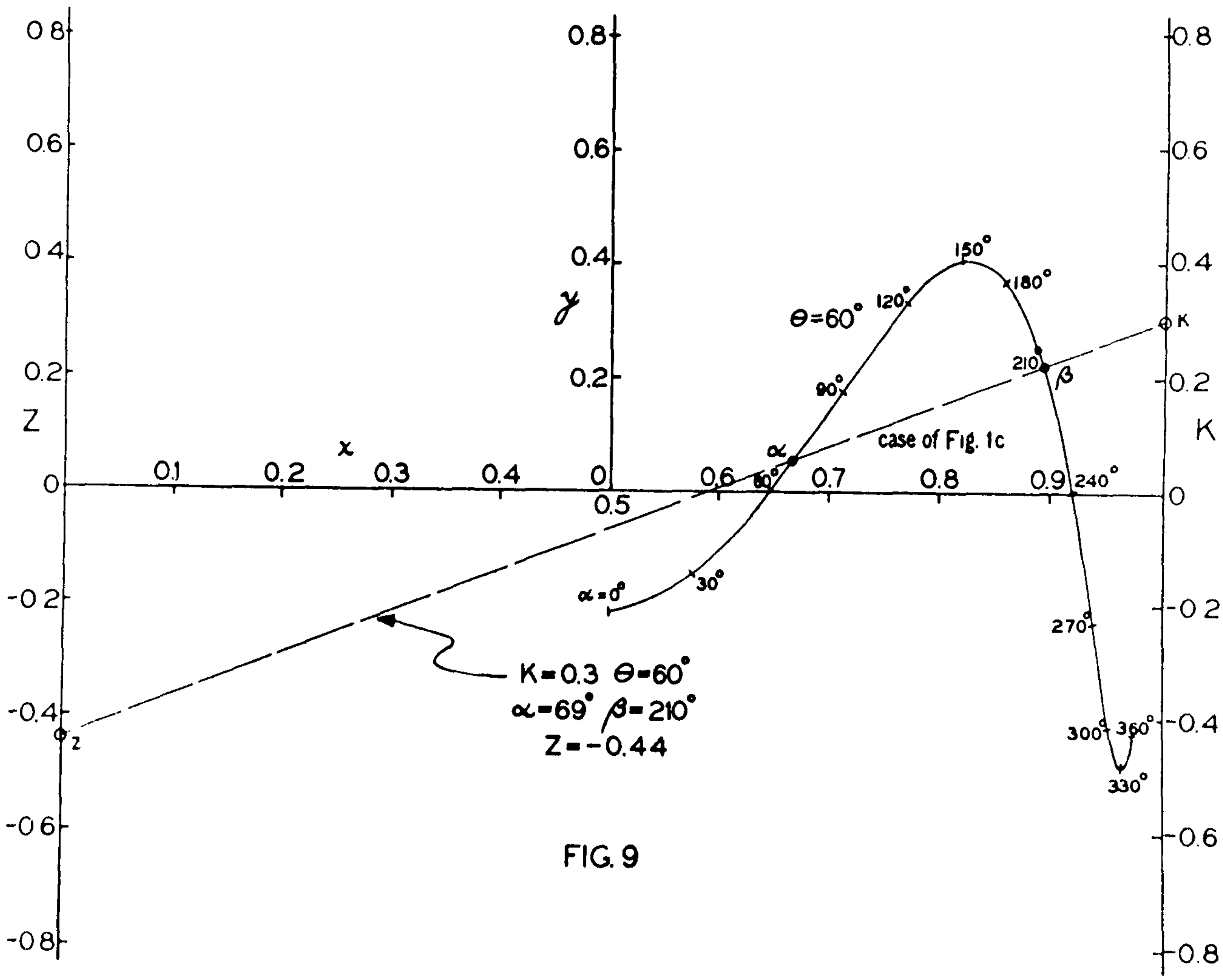

Fig. 9 Deriving the nomogram of Fig. 1b

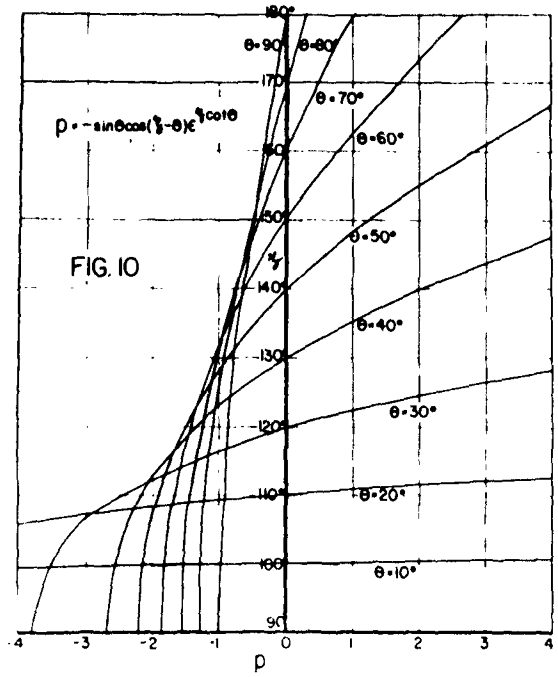

Fig. 10 The quantity p of equation (B-1)

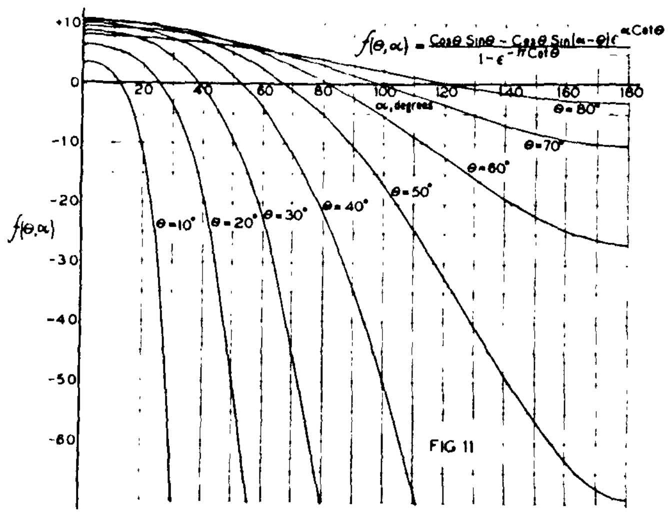

Fig. 11 The right side of equation (B-2)